



Sustainable Energy
**Mod. 6: Fuel Cells & Distributed
Generation Systems**

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Exercise I: fuel cells (calculations)

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Exercise 1: What hydrogen flow rate is required to generate 1.0 ampere of current in a fuel cell ?

Solution:

- For every molecule of hydrogen (H_2) that reacts within a fuel cell, two electrons are liberated at the fuel cell anode: $H_2 \rightarrow 2H^+ + 2e^-$
- One ampere of current is defined as 1 C/sec.
- Moles of electrons per sec.: $M_e = (1 \text{ [C/s]}) / (F \text{ [C/mol]}) = 1.0364 \cdot 10^{-5} \text{ mol/s}$
- Moles of hydrogen per sec.: $M_{H_2} = M_e / 2 = 0.5182 \cdot 10^{-5} \text{ mol/s}$
- Moles of hydrogen per hour: $M_{H_2/h} = 0.01865 \text{ mol/h}$
- Since molar mass of H_2 is: 2.0158 g/mol
- Mass of hydrogen per hour: $M_{H_2/h} = 3.759 \cdot 10^{-5} \text{ kg/h}$

Exercise 2: A 1.0 MW_{DC} fuel cell stack is operated with a cell voltage of 700 mV on pure hydrogen with a fuel utilization, U_f of 80%. (a) How much hydrogen will be consumed in lb/h? (b) What is the required fuel flow rate? (c) What is the required air flow rate for a 25% oxidant utilization, U_{ox} ?

Solution:

- Power (P) is the product of voltage (V) and current (I): $P = I \cdot V$
- So: $I = P/V = 1429 \text{ kA}$
- Mass of H₂ consumed (kg/h): $M_c = (I[A]) \cdot (M_{H_2/h} [\text{kg}/(\text{h} \cdot \text{A})]) = 53.7 \text{ kg/h}$
- Mass of H₂ consumed (lb/h): $M_c = 53.7/0.4536 = 118.4 \text{ lb/h}$
- Fuel flow rate: $H_{2,in} = M_c/U_f = 148.0 \text{ lb/h}$
- Since molar mass of H₂ is: 2.0158 g/mol
- Mass O₂ consumed (mol/h): $M_{O_2} = (M_c \cdot 1000)/(2 \cdot 2.0158) = 13319.8 \text{ mol/h}$
- O₂ flow rate: $O_{2,in} = M_{O_2}/U_{ox} = 53279.2 \text{ mol/h}$
- Since O₂ concentration is 21% and air molar mass is 28.85 g/mol
- Air mass flow rate: $M_{air} = (O_{2,in}/0.21) \cdot 28.85 = 7319547.2 \text{ g/h} = 16136.6 \text{ lb/h}$

Exercise 3: A PAFC, operating on reformed natural gas (900 lb/h) and air, has a fuel and oxidant utilization of 86% and 70%, respectively. With the fuel and oxidant composition and molecular weights (MW) listed below, how much hydrogen will be consumed in mol/h? (b) How much oxygen is consumed in mol/h? (c) What is the required air flow rate in mol/h and lb/h? (d) How much water is generated? (e) What is the composition of the effluent (spent) fuel and air streams in mol percent?

Fuel Data	mol percent
CH ₄	4.0
CO	0.4
CO ₂	17.6
H ₂	75.0
H ₂ O	3.0
Total	100.0
MW	10.55

Air Data	mol percent, dry	mol percent, wet
H ₂ O	0.00	1.00
N ₂	79.09	78.21
O ₂	21.00	20.79
Total	100.00	100.00
MW	28.85	28.74

Solution (1/2):

- Mass of fuel supplied (mol/h): $M_{c_s} = (900 \cdot 453.6) / (10.55) = 38695.7$ mol/h
- Mass of H₂ supplied (mol/h): $M_{H_2_s} = M_{c_s} \cdot 0.75 = 29021.8$ mol/h
- Mass of H₂ consumed (mol/h): $M_{H_2_c} = M_{H_2_s} \cdot 0.86 = 24958.7$ mol/h
- Mass of O₂ consumed (mol/h): $M_{O_2_c} = M_{H_2_c} / 2 = 12479.4$ mol/h
- Mass of O₂ supplied (mol/h): $M_{O_2_s} = M_{O_2_c} / 0.70 = 17827.7$ mol/h
- Mass of air (wet) supplied (mol/h): $M_{air_s} = M_{O_2_s} / 0.2079 = 85751.3$ mol/h
- Mass of air (wet) supplied (lb/h): $M_{air_s} = 85751.3 \cdot 28.74 / 453.6 = 5433.1$ lb/h
- Mass of water generated (mol/h): $M_{H_2O_G} = M_{H_2_c} = 24958.7$ mol/h
- Fuel side:

	mol percent	lb mol/hr			mol percent
Gas	FC inlet	FC inlet	FC reaction	FC outlet	FC outlet
CH ₄	4.0	3.41		3.41	11.27
CO	0.4	0.34		0.34	1.13
CO ₂	17.6	15.01		15.01	49.58
H ₂	75.0	63.97	-55.01	8.96	29.58
H ₂ O	3.0	2.56		2.56	8.45
Total	100.0	85.29	-55.01	30.28	100.00

- In PAFC, only the moles of hydrogen change on the anode side
- These inert gases act to dilute hydrogen lowering cell voltage

Solution (2/2):

• Air side:

Gas	mol percent		lb mol/hr		mol percent
	FC inlet	FC inlet	FC reaction	FC outlet	
H ₂ O	1.00	1.89	55.01	56.90	26.28
N ₂	78.21	147.82		147.82	68.27
O ₂	20.79	39.30	-27.51	11.79	5.44
Total	100.00	189.01	27.51	216.51	100.00

$$M_{\text{air}_s} = 85751.3 / 453.6 = 189.04 \text{ lb mol/h}$$

Exercise 4: An MCFC operating on 1,000 lb/h of fuel gas and a 70% air/30% CO₂ oxidant has a fuel and oxidant utilization of 75% and 50% respectively. With the fuel and oxidant composition and molecular weights listed below, (a) How much hydrogen will be consumed in mol/h? (b) How much oxygen is consumed in mol/h? (c) What are the required air and oxidant flow rates in mol/h? (d) How much CO₂ is transferred from the cathode to the anode? (e) What is the composition of the effluent (spent) fuel and oxidant streams in mol percent (no water gas shift reaction)?

Fuel Data	Mol percent
CH ₄	0.0
CO	0.0
CO ₂	20.0
H ₂	80.0
H ₂ O	0.0
Total	100.0
MW	10.42

Oxidant Data	Air	Air + CO ₂
	mol percent, wet	Mol percent, wet
CO ₂	0.00	30.00
H ₂ O	0.00	0.70
N ₂	78.21	54.75
O ₂	20.79	14.55
Total	100.00	100.00
MW	28.74	33.32

Solution (1/2):

- Mass of fuel supplied (mol/h): $M_{c_s} = (1000 \cdot 453.6) / (10.42) = 43531.67 \text{ mol/h}$
- Mass of H_2 supplied (mol/h): $M_{H_2_s} = M_{c_s} \cdot 0.80 = 34825.3 \text{ mol/h}$
- Mass of H_2 consumed (mol/h): $M_{H_2_c} = M_{H_2_s} \cdot 0.75 = 26119.0 \text{ mol/h}$
- Mass of O_2 consumed (mol/h): $M_{O_2_c} = M_{H_2_c} / 2 = 13059.5 \text{ mol/h}$
- Mass of air (wet) supplied: $M_{air_s} = ((M_{O_2_c} / 0.50) / 0.2079) = 125632.5 \text{ mol/h}$
- Mass of oxidant supplied (mol/h): $M_{ox_s} = M_{air_s} / 0.70 = 179475.0 \text{ mol/h}$
- Since: $H_{2, \text{anode}} + \frac{1}{2} O_{2, \text{cathode}} + CO_{2, \text{cathode}} \rightarrow H_2O_{, \text{anode}} + CO_{2, \text{anode}}$
- Mass of CO_2 transferred=consumed (mol/h): $M_{CO_2_t} = M_{H_2_c} = 26119.0 \text{ mol/h}$
- Fuel side:

Gas	mol percent		lb mol/hr		Mol percent	
	FC inlet	FC inlet	FC reaction	FC outlet	FC outlet	FC outlet
CH ₄	0.0	0.00		0.00	0.00	0.00
CO	0.0	0.00		0.00	0.00	0.00
CO ₂	20.0	19.20	57.61	76.82	50.00	50.00
H ₂	80.0	76.82	-57.61	19.20	12.50	12.50
H ₂ O	0.0	0.00	57.61	57.61	37.50	37.50
Total	100.0	96.02	-57.61	153.63	100.00	100.00

- in the MCFC, both oxygen and carbon dioxide are consumed on the cathode (air) side.

Solution (2/2):

• Air side:

Gas	mol percent		lb mol/hr		Mol percent
	FC inlet	FC inlet	FC reaction	FC outlet	FC outlet
CO ₂	30.00	83.13	-57.61	25.52	13.38
H ₂ O	0.70	1.94		1.94	1.02
N ₂	54.70	151.71		151.71	79.56
O ₂	14.6	40.33	-28.81	11.52	6.04
Total	100.00	277.11	-86.42	190.69	100.00

$$M_{\text{air}_s} = 125632.5 / 453.6 = 277 \text{ lb mol/h}$$

Exercise 5: Given a desired output of $2.0 \text{ MW}_{\text{DC}}$ and the desired operating point of 600 mV and 400 mA/cm^2 , (a) How much fuel cell area is needed? (b) Assuming a cell area of 1.00 m^2 per cell and 280 cells per stack, how many stacks are needed for this 2.0 MW unit?

Solution:

- Total current: $I = P/V = 2 \cdot 10^6 / 0.6 = 3,333 \text{ kA} = 3.333 \text{ MA}$
- Cell area: $A = I/\text{Current density} = 3.333 \cdot 10^6 / 0.4 = 8,333,333 \text{ cm}^2$
- Number of cells: $N_{\text{cell}} = A/10000 = 833 \text{ cells}$
- Number of stacks: $N_{\text{stack}} = N_{\text{cell}}/280 = 2.98 \text{ stacks} \approx 3 \text{ stacks}$